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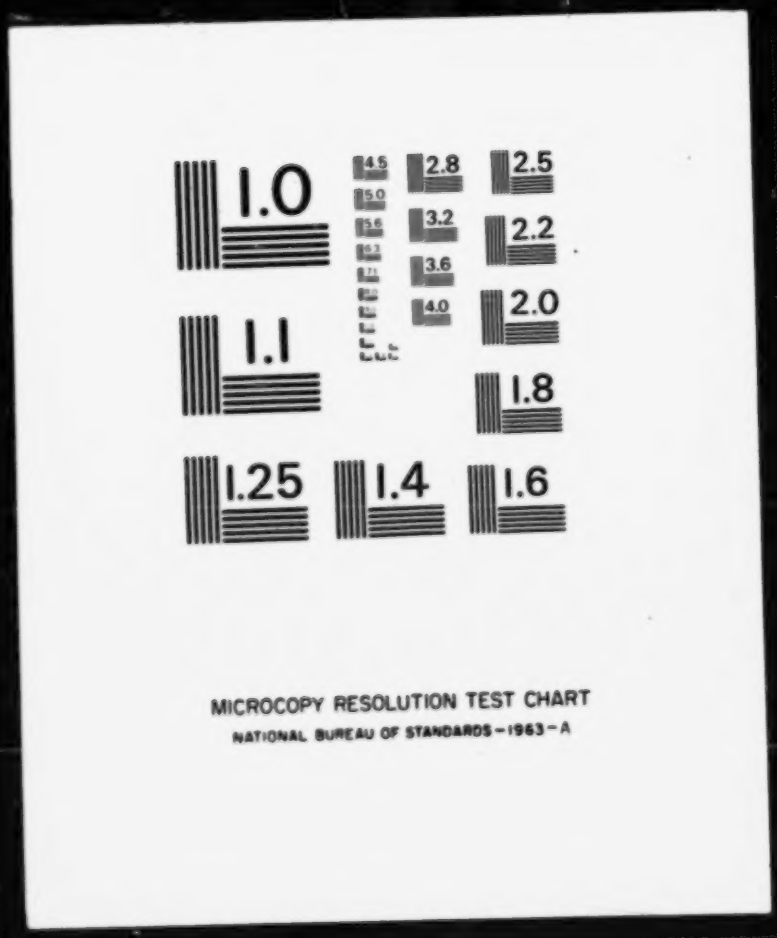

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This programed mathematics textbook is for student use in vocational education courses. It was developed as part of a programed series covering 21 mathematical competencies which were identified by university researchers through task analysis of several occupational clusters. The development of a sequential content structure was also based on these mathematics competencies. After completion of this program the student should be able to: (1) change from exponential form to expanded form and vice versa, (2) write a number in the base 10 system in expanded exponential form, (3) write a number in the base two system in expanded exponential form, and (4) convert numbers from base two and base five to base 10. The material is to be used by individual students under teacher supervision. Twenty-six other programed texts and an introductory volume are available as VT 006 882-VT 006 909, and VT 006 975. (EM)

FINAL REPORT
Project No. OE7-0031
Contract No. OEG-4-7-070031-1626
Report No. 16-U

# Occupational Mathematics

# CONCEPTS OF NUMBER BASES

June 1968

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U.S. DEPARTMENT OF
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Occupational Mathematics

# CONCEPTS OF NUMBER BASES

Project No. 0E7-0031
Contract No. OEG-4-7-070031-1626
Report No. 16-U

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June 1968

The research reported herein was performed pursuant to a contract with the Office of Education, U.S. Department of Health, Education, and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their professional judgment in the conduct of the project. Points of view or opinions stated do not, therefore, necessarily represent official Office of Education position or policy.

Washington State University, Department of Education, Pullman, Washington
State Coordinating Council for Occupational Education, Olympia, Washington

# OBJECTIVES

1. The student should be able to change from exponential form to expanded form and vice versa.

2. The student should be able to write a number in the base 10 system in expanded exponential form.

3. The student should be able to write a number in the base 2 system in expanded exponential form.

4. The student should be able to convert numbers from base 2 and base 5 to base 10.

Greetings! You are about to begin improving your knowledge of basic mathematics. There are many important uses for the mathematics you are learning.

This booklet is not like your ordinary books. It is designed to help you learn as an individual. On the following pages you will find some information about mathematics. After the information is presented, you will be asked a question. Your answers to these questions will determine how you proceed through this booklet. When you have selected your answer to the question, turn to the page you are told to.

Do not write in this booklet. You may wish to have a pencil and some paper handy so you can write when you want to.

Remember this is not an ordinary book.

1. Study the material on the page.
2. Read the question on the page (you may want to restudy the material on the page).
3. Select the answer you believe is correct.
4. Turn to the page indicated by your answer.

Are you ready to begin?

(a) Yes — Turn to page 1

(b) No — Turn to page C

(c) HELP — Go see your teacher

Your answer was (b) No.

Well, this booklet is a little different.

Go back and read page B again. After you have read it, you will probably be ready to begin.

In order to understand the main ideas about numbers of different bases, we must know a little about exponents.

Now, an exponent is merely a method for showing repeated multiplications. For example, instead of writing 2 x 2 x 2 x 2, we just write $2^4$ ($2^4$ is read as 2 to the fourth power).

What does $5^3$ mean?

(a) 3 x 5 Turn to page 5

(b) 5 x 5 x 5 Turn to page 3

No! Wrong answer!

Return to page 1 and begin this Unit again. Read the material carefully before choosing an answer.

5 x 5 x 5 is correct!

Now, how would you write a x a x a x a in exponential form?

(a) $a_4$ Turn to page 6

(b) 4a Turn to page 10

(c) $a^4$ Turn to page 8

Incorrect. $4 \times 7 \neq 7 \times 7 \times 7 \times 7$.

Go to page 7 and work the problem again.

Incorrect.

Five to the third power means that we are going to multiply 3 five's together. This is written as 5 x 5 x 5.

How would you write $10^5$?

(a) 10 x 10 x 10 x 10 x 10 Turn to page 7

(b) 5 x 10 Turn to page 2

Whoops! Any number written in the lower right hand corner is called a subscript, not an exponent.

Go to page 3 and choose a different answer.

10 x 10 x 10 x 10 x 10 is the correct answer!

Now, how would you write 7 x 7 x 7 x 7 in exponent form?

| | | |
|---|---|---|
| (a) | 4 x 7 | Turn to page 4 |
| (b) | $4^7$ | Turn to page 2 |
| (c) | $7^4$ | Turn to page 3 |
| (d) | 47 | Turn to page 13 |

Very good!

Try this problem.

In exponential form, 1/10 x 1/10 x 1/10 = ___?___ .

(a) 1/1000 Turn to page 9

(b) $(\frac{1}{10})^3$ Turn to page 11

(c) $\frac{1}{10}3$ Turn to page 17

(d) 3/10 Turn to page 21

While 1/1000 is the correct <u>product</u> of 1/10 x 1/10 x 1/10, it is <u>NOT</u> the exponential form of 1/10 x 1/10 x 1/10.

What is 1/5 x 1/5 x 1/5 x 1/5 in exponential form?

(a) $(1/5)^4$ Turn to page 20

(b) 4 x 1/5 Turn to page 21

Your answer is incorrect.

4a means 4 x a, not a x a x a x a.

Let's consider an example to show you why your answer was not correct. Let a = 3, then 4a = 4 x 3 = 12, while a x a x a x a = 3 x 3 x 3 x 3 = 81. Quite a difference, isn't there?

Try this problem now.

10 x 10 x 10 in exponential form is:

(a) 1000 Turn to page 15

(b) $10^3$ Turn to page 18

(c) 3 x 10 Turn to page 21

$(1/10)^3$ is correct!

Now for a very important question.

Does $(1/10)^3 = \frac{1}{10}3$ ?

(a) Yes Turn to page 12

(b) No Turn to page 14

Okay, let's continue with this question.

$\frac{1}{6}4 =$ ? .

(a) 1/24 Turn to page 25

(b) 1/1296 Turn to page 22

(c) 24 Turn to page 19

Come now! $7 \times 7 \times 7 \times 7 \neq 47$. In fact, $7 \times 7$ is 49 and that is larger than 47 without multiplying by the other seven's.

Go to page 7 and make a better choice this next time.

Whoops!

$(1/10)^3 = 1/10 \times 1/10 \times 1/10 = 1/1000.$

$\frac{1}{10}3 = \frac{1}{10 \times 10 \times 10} = 1/1000.$

Thus, $(1/10)^3$ <u>does equal</u> $\frac{1}{10}3$.

Turn to page 12 and continue.

While 1000 is equal to 10 x 10 x 10, it is <u>not</u> the exponential form of 10 x 10 x 10.

Return to page 10 and make a more appropriate selection.

You seem to be having trouble multiplying fractions.

Go to Unit 6 and review the concepts presented there.

Then, upon completion, return to page 1 of this Unit.

$\frac{1}{10}3$ is correct.

Now for a very important question.

Does $\frac{1}{10}3 = (1/10)^3$?

(a) Yes Turn to page 12

(b) No Turn to page 14

Correct!

Continue with this problem.

Write $x^4$ in expanded form.

(a) 4x Turn to page 21

(b) (x)(x)(x)(x) Turn to page 8

Ooops!  You forgot the definition of an exponent.

Return to page 1 and begin again.

$(1/5)^4$ is correct.

Now go to page 8 and continue.

No! Wrong answer!

Return to page 1 and begin this Unit again. Read the material carefully before choosing an answer.

You are doing fine! Let's continue.

In some problems we will see a negative exponent, i.e., $3^{-2}$. The minus sign in front of the exponent indicates that we want the reciprocal of the given number. For example, $3^{-2}$ means the RECIPROCAL of $3^2$.

What is the reciprocal of $3^2$? It is $\frac{1}{3}2$. (Note: If this idea gives you trouble, you should go work Unit 17 on Reciprocals before continuing.)

Now answer this question.

Which of the following answers is correct for $x^{-3}$?

(a) $x^3$ — Turn to page 28

(b) $\frac{1}{3x}$ — Turn to page 31

(c) $\frac{1}{(x)(x)(x)}$ — Turn to page 26

Okay! 1/9 is correct!

Work this problem.

$(1/x)(1/x)(1/x) = ?$

(a) $1/3x$ Turn to page 21

(b) $3(1/x)$ Turn to page 19

(c) $\frac{1}{x}3$ Turn to page 22

Incorrect.

$10^{-4}$ means you want the RECIPROCAL of $10^4$ which is $\frac{1}{10^4}$.

Which answer below is equal to $5^{-1}$?

(a) 1/5 Turn to page 36

(b) 5 Turn to page 35

(c) -5 Turn to page 33

No! $\frac{1}{6}4 = (1/6)^4 = 1/6 \times 1/6 \times 1/6 \times 1/6 = 1/1296.$

What does $\frac{1}{3}2$ equal?

(a) 1/9 Turn to page 23

(b) 1/6 Turn to page 27

(c) 2/3 Turn to page 19

Very good!

Continue with this question.

Which answer below is equal to $10^{-4}$?

(a) $\frac{1}{10}4$ Turn to page 29

(b) 1/40 Turn to page 24

(c) 10 x 10 x 10 x 10 Turn to page 32

Wait a minute!

$\frac{1}{3}2$ means $(1/3)^2$ or 1/3 x 1/3.

Now, what is 1/3 x 1/3?

| | | |
|---|---|---|
| (a) | 1/6 | Turn to page 16 |
| (b) | 1/9 | Turn to page 23 |
| (c) | 2/3 | Turn to page 21 |

No!

$x^{-3}$ is the <u>reciprocal</u> of $x^3$.

What answer below is correct for $10^{-3}$?

| | | |
|---|---|---|
| (a) | 1/30 | Turn to page 33 |
| (b) | $\frac{1}{10}3$ | Turn to page 30 |
| (c) | $10^3$ | Turn to page 35 |

Excellent! Keep up the good work.

There is one more type of exponent that needs to be explained. This is when the exponent is zero. ANYTHING to the zero power is 1. For example, $a^0 = 1$, $10^0 = 1$, $(25 \times 13 \times bc)^0 = 1$, etc.

Maybe the entire sequence of exponents will fall into place if we look at the numbers from $10^3$ to $10^{-3}$.

Thus, $10^3$, $10^2$, $10^1$, $10^0$, $10^{-1}$, $10^{-2}$, $10^{-3}$ written without exponents are 1000, 100, 10, 1, 1/10, 1/100, 1/1000.

Now, what does $3^0$ equal?

(a) 0 Turn to page 43

(b) 1 Turn to page 41

(c) 3 Turn to page 39

That's correct!

Which answer below is equivalent to $x^{-1}$?

(a) x — Turn to page 35

(b) -1x — Turn to page 33

(c) 1/x — Turn to page 26

No!

$x^{-3}$ is the <u>reciprocal</u> of $x^3$.

What answer below is correct for $10^{-3}$?

| | | |
|---|---|---|
| (a) | 1/30 | Turn to page 33 |
| (b) | $\frac{1}{10}3$ | Turn to page 30 |
| (c) | $10^3$ | Turn to page 35 |

Incorrect.

$10^{-4}$ means you want the RECIPROCAL of $10^4$ which is

$\frac{1}{10^4}$.

Which answer below is equal to $5^{-1}$?

(a) 1/5 Turn to page 36

(b) 5 Turn to page 35

(c) -5 Turn to page 33

Your answer is incorrect.

Are you having trouble with reciprocals?

What is the <u>reciprocal</u> of $5^3$?

(a) 125 Turn to page 34

(b) 1/125 Turn to page 30

Incorrect.

In order to give you a better understanding of reciprocals, go to Unit 17 and work through that unit. Then return to page 22 of this unit.

Your answer is incorrect.

Are you having trouble with reciprocals?

What is the <u>reciprocal</u> of $5^3$?

(a) 125 Turn to page 34

(b) 1/125 Turn to page 30

Your answer is correct.

Try this problem.

1/8 is equal to which of the following? (Be careful now.)

(a) $\frac{1}{2}$-3 Turn to page 38

(b) $4^{-2}$ Turn to page 37

(c) $2^{-3}$ Turn to page 29

Oops! $4^2 = 16$, not 8.

Return to page 36 and make a better choice next time.

Your answer is incorrect.

Just keep in mind that the <u>minus</u> sign in <u>front</u> of the exponent denotes a RECIPROCAL. Hence $\frac{1}{2}^{-3}$ equals the RECIPROCAL of $(1/2)^3$ or 8.

Now, 1/2 is equal to which answer below?

(a) $2^{-1}$ Turn to page 36

(b) $2^{1}$ Turn to page 33

No!

Any number raised to the ZERO power is 1.

What does $a^0$ equal?

(a) a — Turn to page 42

(b) 1 — Turn to page 45

No!

Anything to the ZERO power is 1.

Return to page 44 and try again.

Very good! Any number of quantity to the zero power is 1.

Work this one now.

$10x^0$ equals:

(a) 10 Turn to page 48

(b) 10x Turn to page 46

(c) 1 Turn to page 44

That zero exponent is giving you trouble. It isn't that hard.

Return to page 29 and begin this section again. Take your time and read the material carefully before selecting your answers.

Turn to page 29.

No!

Any number raised to the ZERO power is 1.

What does $a^0$ equal?

(a) a — Turn to page 42

(b) 1 — Turn to page 45

Oops!

$10x^0$ means 10 times $x^0$. Now $x^0 = 1$, so $10x^0 = 10(x^0) = 10(1) = 10$.

What does $(5x)^0$ equal?

(a) 5 Turn to page 49

(b) 5x Turn to page 40

(c) 1 Turn to page 47

1 is correct!

What does $(3a)^0$ equal?

(a) 3 Turn to page 42

(b) 3a Turn to page 39

(c) 1 Turn to page 41

Oops!

$10x^0$ means 10 times $x^0$. Now, $x^0 = 1$, so $10x^0 = 10(x^0) = 10(1) = 10$.

What does $(5x)^0$ equal?

(a) 5 Turn to page 49

(b) 5x Turn to page 40

(c) 1 Turn to page 47

Fine!  1 is correct.

What does $5x^0$ equal?

(a) 5 Turn to page 48

(b) 1 Turn to page 50

Very good! Your answer is correct.

What does $2^4$ equal?

(a) 16 Turn to page 53

(b) 8 Turn to page 55

Your answer is incorrect.

$(5x)^0$ means that the quantity 5x is raised to the zero power.

What does $(10x^2)^0$ equal?

(a) 10 Turn to page 42

(b) 1 Turn to page 47

(c) $10x^2$ Turn to page 39

Whoops!

$5x^0$ means 5 times $x^0$.

Now, what is 5 times $x^0$?

(a) 5x — Turn to page 43

(b) 0 — Turn to page 39

(c) 1 — Turn to page 42

(d) 5 — Turn to page 48

Right! $3^4 = 81$.

Which of the following is equal to 16?

(a) $2^8$ Turn to page 54

(b) $4^2$ Turn to page 53

(c) $8^2$ Turn to page 56

Your answer is wrong.

The minus sign in front of the exponent indicates a RECIPROCAL. Remember? Therefore, $3^{-3}$ is the reciprocal of $3^3$.

The reciprocal of $3^3$ is:

(a) 1/27 Turn to page 57

(b) 1/9 Turn to page 59

Correct!

What does $3^{-3}$ equal?

(a) 27 Turn to page 52

(b) 1/9 Turn to page 58

(c) 1/27 Turn to page 60

Oops! You forgot the definition of an exponent.

Return to page 1 and begin again.

Incorrect!

$2^4$ = 2 x 2 x 2 x 2, which is 16.

What does $3^4$ equal?

(a) 81 Turn to page 51

(b) 12 Turn to page 54

(c) 64 Turn to page 56

No! No! No! .

An exponent indicates repeated multiplication.

Examples: $4^3$ means 4 x 4 x 4, $2^4$ = 2 x 2 x 2 x 2,

etc.

Now return to page 55 and work the problem on that page.

Okay!

Now, what does $3^{-3}$ equal?

(a) 1/9 Turn to page 59

(b) 1/27 Turn to page 60

(c) 27 Turn to page 33

Your answer is wrong.

The minus sign in front of the exponent indicates a RECIPROCAL. Remember? Therefore, $3^{-3}$ is the reciprocal of $3^3$.

The reciprocal of $3^3$ is:

(a) 1/27 Turn to page 57

(b) 1/9 Turn to page 59

Wait a minute!

The reciprocal of $3^3$ is $\frac{1}{3}3$ which is equal to $\frac{1}{3 \times 3 \times 3}$ or 1/27.

What is the reciprocal of $5^3$?

(a) 125 Turn to page 34

(b) 1/15 Turn to page 56

(c) 1/125 Turn to page 57

In order to understand how numbers can be written in different bases, we must know how the numbers we use can be expressed in terms of exponents and what these numbers really mean.

Consider the number 314. It is really the sum of 300 + 10 + 4. We write it as 314 to save time, paper, and work. We could and sometimes should write 314 in tabular form like:

| hundreds | tens | units |
|---|---|---|
| 3 | 1 | 4 |

which shows us that we have 3 hundreds, one ten, and 4 units. In order to show this relationship, we sometimes write in EXPANDED EXPONENTIAL FORM as follows:

$$314 = 300 + 10 + 4$$
$$= 3 \times 100 + 1 \times 10 + 4 \times 1$$
$$= 3 \times 10^2 + 1 \times 10^1 + 4 \times 10^0$$

$3 \times 10^2 + 1 \times 10^1 + 4 \times 10^0$ is 314 in <u>expanded exponential form</u>.

Turn to page 61

Write 1523 in expanded exponential form:

(a) $1000 + 500 + 20 + 3$

Turn to page 65

(b) $1 \times 1000 + 5 \times 100 + 2 \times 10 + 3 \times 1$

Turn to page 68

(c) $1 \times 10^3 + 5 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$

Turn to page 70

Incorrect!

Expanded exponential form requires that our expanded form be equal to the number from which it came. For example, $4 \times 10^4 + 5 \times 10^3 + 7 \times 10^2 + 2 \times 10^1 + 8 \times 10^0 = 40000 + 5000 + 70 + 20 + 8 = 45{,}728$.

Keeping this example in mind, which of the following is true?

(a) $4020 = 5 \times 10^3 + 0 \times 10^2 + 2 \times 10^0$
Turn to page 71

(b) $1030 = 1 \times 10^3 + 0 \times 10^2 + 3 \times 10^1 + 0 \times 10^0$
Turn to page 67

(c) $2003 = 2 \times 10^1 + 3 \times 10^0$
Turn to page 80

(d) $305 = 3 \times 10^2 + 5 \times 10^0$ Turn to page 70

That's correct!

Now write 2003 in expanded exponential form.

(a) $2 \times 10^1 + 3 \times 10^0$ Turn to page 80

(b) $2 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 + 3 \times 10^0$
Turn to page 75

(c) 2000 + 3 Turn to page 65

Correct!

If $10,010 = 1 \times 2^4 + 1 \times 2^1$, then 10,010 is written in base:

(a) 2 Turn to page 73

(b) 10 Turn to page 85

Incorrect.

While expanded exponential form is writing our number as a sum of multiples of 10, it is still not in its correct form until we have the multiples of 10 written in EXPONENTIAL form. For example,

$100 = 1 \times 10^2$

$3000 = 3 \times 1000 = 3 \times 10^3$

$2 = 2 \times 1 = 2 \times 10^0$

$25 = 20 + 5 = 2 \times 10 + 5 \times 1 = 2 \times 10^1 + 5 \times 10^0$

Now, which is in expanded exponential form?

(a) $300 + 50 + 2$ Turn to page 71

(b) $3 \times 100 + 5 \times 10 + 2 \times 1$ Turn to page 72

(c) $3 \times 10^2 + 5 \times 10^1 + 2 \times 10^0$ Turn to page 63

No, no, no!

$5 \times 10^7$ means 5 times 10 to the seventh power.

Therefore, the EXPONENT is 7 and shows us that the base number is:

(a) 10 Turn to page 64

(b) 5 Turn to page 78

Correct!

2,003 written in expanded exponential form is:

(a) $2 \times 10^3 + 3 \times 10^0$ Turn to page 75

(b) $2 \times 10^3 + 0 \times 10^2 + 1 \times 10^1 + 3 \times 10^0$
Turn to page 62

(c) 2,000 + 3 Turn to page 65

While it is correct to write 1523 as 1 x 1000 + 5 x 100 + 2 x 10 + 3 x 1, it still is not in expanded exponential form. You should know this since there are no exponents on any of the numbers.

Which of the following is in expanded exponential form?

(a) $3 \times 100 + 4 \times 10 + 6 \times 1$ Turn to page 72

(b) $3 \times 10^2 + 4 \times 10^1 + 6 \times 10^0$ Turn to page 63

Incorrect.

The subscript tells you what base you are using.

Return to page 82 and make a different selection.

That's correct!

Now write 2003 in expanded exponential form.

(a) $2 \times 10^1 + 3 \times 10^0$ Turn to page 80

(b) $2 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 + 3 \times 10^0$
Turn to page 75

(c) 2000 + 3 Turn to page 65

Incorrect.

Return to page 60 and continue from there.

Wrong choice.

A number like 3 x 1000 + 2 x 100 + 4 x 10 + 3 x 1 is NOT in exponential form. The reason is obvious. 1000, 100, 10, and 1 are not written as 10 to some exponent.

Be more careful and try this problem.

Which of the following is written in expanded exponential form?

(a) $4 \times 10^3 + 3 \times 10^1$ Turn to page 63

(b) 4000 + 30 Turn to page 71

(c) 4 x 1000 + 0 x 100 + 3 x 10 + 0 x 1 Turn to page 65

Correct!

Let's continue.

The number $1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$ is written in Base:

(a) 10 Turn to page 85

(b) 5 Turn to page 77

(c) 2 Turn to page 82

Incorrect.

The problem is in Base 10 if NO subscript is written.
This is because we use base 10 the most, and it is
too much trouble to write the subscript every time.

Return to page 79 and make another choice.

Very good! $2 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 + 3 \times 10^0$ is correct.

We hope you have noticed that the number which has the exponent determines the place or the column in which the number belongs. Thus $2 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 + 3 \times 10^0$ is

| $10^3$ | $10^2$ | $10^1$ | $10^0$ |
|---|---|---|---|
| 2 | 0 | 0 | 3 |

or as we usually write it, 2003.

This number with the exponent is called the Base of the number. So far, the numbers we have been using have been to the Base 10.

Turn to page 76

If we had a number written in the expanded exponential form of $4 \times 5^2 + 3 \times 5^1 + 1 \times 5^0$, it would be written in Base:

(a) 5 Turn to page 73

(b) 10 Turn to page 78

(c) 2 Turn to page 81

(d) 4 Turn to page 85

What???????

Where did the 5 come from?

Return to page 73 and make a different selection.

Incorrect.

The number with the exponent is the base number.

$3 \times 4^3 + 2 \times 4^1 = 3020$ in Base:

(a) 2 Turn to page 85

(b) 3 Turn to page 81

(c) 4 Turn to page 64

That's correct!

What base is 532 written in?

(a) 5 Turn to page 74

(b) 2 Turn to page 88

(c) 10 Turn to page 90

Oops!

$2 \times 10^1 + 3 \times 10^0 = 20 + 3 = 23$. You were supposed to write 2003 in exponential form, not 23.

What number is $4 \times 10^4 + 9 \times 10^2 + 5 \times 10^1 + 2 \times 10^0$ equal to?

(a) 4952 Turn to page 62

(b) 40,952 Turn to page 67

(c) 4,090,502 Turn to page 83

Wrong choice.

The base number is the number with the exponent on it, when written <u>in</u> expanded exponential form.

The number $5 \times 10^7 + 3 \times 10^5 + 2 \times 10^1$ is written in base:

(a) 5 Turn to page 66

(b) 10 Turn to page 64

Very good! Your answer was correct.

To show that numbers are written to different bases, a number (called a subscript) is written in the lower right hand corner by the number. For example, $3201_5$ means that 3201 is written to the base 5 and is designated by the subscript 5. (Note: If no subscript is shown, then the number is written in base 10.)

$10,101_2$ means that 10,101 is written in base:

(a) 1 Turn to page 69

(b) 2 Turn to page 79

(c) 10 Turn to page 84

Oops!

Much too big a number.

Return to page 80 and make a better choice.

Incorrect.

The subscript tells you what base you are using.

Return to page 82 and make a different selection.

Wrong choice.

The base number is the number with the exponent on it, when written <u>in</u> expanded exponential form.

The number $5 \times 10^7 + 3 \times 10^5 + 2 \times 10^1$ is written in base:

(a) 5 Turn to page 66

(b) 10 Turn to page 64

That's correct!

Write $4032_5$ in expanded exponential form.

(a) $4 \times 10^3 + 0 \times 10^2 + 3 \times 10^1 + 2 \times 10^0$
Turn to page 89

(b) $4 \times 2^3 + 0 \times 2^2 + 3 \times 2^1 + 2 \times 2^0$
Turn to page 91

(c) $4 \times 5^2 + 3 \times 5^1 + 2 \times 5^0$
Turn to page 94

(d) $4 \times 5^3 + 0 \times 5^2 + 3 \times 5^1 + 2 \times 5^0$
Turn to page 97

No!

Our problem is in base 2. Let's be more careful when we read and work these.

Return to page 94 and work the problem there.

Incorrect.

The problem is in Base 10 if NO subscript is written.
This is because we use base 10 the most, and it is
too much trouble to write the subscript every time.

Return to page 79 and make another choice.

Incorrect.

The subscript of 5 indicates that we are working in base 5. Therefore, the number with the exponent on it should be a 5.

Return to page 86 and work that problem again.

Your answer is correct!

Now write $1,101_2$ in expanded exponential form.

(a) $1 \times 10^3 + 1 \times 10^2 + 1 \times 10^0$ Turn to page 95

(b) $1000 + 100 + 1$ Turn to page 92

(c) $1 \times 2^3 + 1 \times 2^2 + 1 \times 2^0$ Turn to page 86

Incorrect.

The subscript of 5 indicates that we are working in base 5. Therefore, the number with the exponent on it should be a 5.

Return to page 86 and work that problem again.

No!

$1,101_2$ is in base 2, not base 10.

Return to page 90 and try again.

Wrong choice.

$10,101_2$ is written in expanded exponential form as

$1 \times 2^4 + 0 + 1 \times 2^2 + 0 + 1 \times 2^0$

or

$16 + 0 + 4 + 0 + 1$ or 21.

Try this one.

$20,120_3$ is the number _____ in base 10.

(a) 22 Turn to page 106

(b) 1,285 Turn to page 109

(c) 96 Turn to page 112

Your answer is incorrect.

The exponent on the 5 tells you what column the coefficient belongs. Thus, $4{,}032_5$ is written in tabular form like this:

| $5^3$ | $5^2$ | $5^1$ | $5^0$ |
|---|---|---|---|
| 4 | 0 | 3 | 2 |

or like this: $4 \times 5^3 + 0 \times 5^2 + 3 \times 5^1 + 2 \times 5^0$.

Now write $10{,}101_2$ in expanded exponential form.

(a) $1 \times 10^4 + 0 \times 10^3 + 1 \times 10^2 + 0 \times 10^1 + 1 \times 10^0$
Turn to page 87

(b) $1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
Turn to page 96

(c) $1 \times 5^4 + 0 \times 5^3 + 1 \times 5^2 + 0 \times 5^1 + 1 \times 5^0$
Turn to page 98

(d) $1 \times 2^3 + 0 + 1 \times 2^2 + 0 + 1 \times 2$
Turn to page 100

No!

$1,101_2$ is in base 2, not base 10.

Return to page 90 and try again.

That is correct! Let's continue.

Since most of our work is done in base 10, we often wish to convert from other bases to base 10. Let's look at an example to see how it's done.

$10,111_2$ is a number written to base 2. Written in expanded exponential form, it is $1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$. We now evaluate each term and add them up.

$1 \times 2^4 = 1 \cdot 16 = 16$

$0 \times 2^3 = 0 \cdot 8 = 0$

$1 \times 2^2 = 1 \cdot 4 = 4$

$1 \times 2^1 = 1 \cdot 2 = 2$

$1 \times 2^0 = 1 \cdot 1 = 1$

total 23 Therefore, $10,111_2 = 23$.

Let's see if you can convert $10,101_2$ to base 10.

(a) 21 Turn to page 99

(b) 11 Turn to page 93

(c) 3 Turn to page 101

That is correct! Let's continue.

Since most of our work is done in base 10, we often wish to convert from other bases to base 10. Let's look at an example to see how it's done.

$10,111_2$ is a number written to base 2. Written in expanded exponential form, it is $1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$. We now evaluate each term and add them up.

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$1 \times 2^0 = 1 \cdot 1 = 1$

total 23 Therefore, $10,111_2 = 23$.

Let's see if you can convert $10,101_2$ to base 10.

(a) 21 Turn to page 99

(b) 11 Turn to page 93

(c) 3 Turn to page 101

No!

Our problem is in base 2. Let's be more careful when we read and work these.

Return to page 94 and work the problem there.

Okay, 21 was correct!

Write $4,321_5$ in base 10.

(a) 81 Turn to page 103

(b) 586 Turn to page 105

(c) 211 Turn to page 108

Incorrect.

$10,101_2$ is written in tabular form as:

| $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 1 |

. Thus, in expanded exponential form we have $1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$, or simply $1 \times 2^4 + 1 \times 2^2 + 1 \times 2^0$.

Now write $10,010_2$ in expanded exponential form.

(a) $1 \times 2^4 + 1 \times 2$ Turn to page 86

(b) $1 \times 2^1 + 0 + 0 + 1 \times 2^0 + 0$ Turn to page 104

(c) $1 \times 10^4 + 0 + 0 + 1 \times 10^1 + 0$ Turn to page 98

Wrong choice.

$10,101_2$ is written in expanded exponential form as

$1 \times 2^4 + 0 + 1 \times 2^2 + 0 + 1 \times 2^0$ or

16 + 0 + 4 + 0 + 1 or 21.

Try this one.

$20,120_3$ is the number _____ in base 10.

(a) 22 Turn to page 106

(b) 1,285 Turn to page 109

(c) 96 Turn to page 112

Your answer is incorrect.

The first thing we must do before we can convert to base 10 is to write our problem in expanded exponential form.

$632_7$ written in this form is:

(a) $7 \times 6^2 + 7 \times 3^1 + 7 \times 2^0$ Turn to page 118

(b) $6 \times 7^2 + 3 \times 7^1 + 2 \times 7^0$ Turn to page 107

Sorry, wrong answer.

$4,321_5$ =

| $5^3$ | $5^2$ | $5^1$ | $5^0$ |
|---|---|---|---|
| 4 | 3 | 2 | 1 |

= $4 \times 5^3$ or 500

$3 \times 5^2$ or 75

$2 \times 5^1$ or 10

$1 \times 5^0$ or 1

total 586

Try this one: $632_7$

(a) 317 Turn to page 113

(b) 32 Turn to page 111

(c) 572 Turn to page 102

That's incorrect.

Be careful of those exponents.

Rework the problem on page 100.

Very good!

Work this one.

$2{,}012_3$ is the number _____ in base 10.

(a) 21 Turn to page 114

(b) 59 Turn to page 115

(c) 23 Turn to page 117

Incorrect.

We are working with base 3. You can tell by the subscript, remember?

Return to page 93 and try again.

That's correct!

Now let's turn to page 108 and finish the problem by converting $632_7$ to base 10.

Sorry, wrong answer.

$4,321_5 = \frac{5^3 \mid 5^2 \mid 5^1 \mid 5^0}{4 \mid 3 \mid 2 \mid 1}$

$= 4 \times 5^3$ or 500

$3 \times 5^2$ or 75

$2 \times 5^1$ or 10

$1 \times 5^0$ or 1

total 586

Try this one: $632_7$

(a) 317 Turn to page 113

(b) 32 Turn to page 111

(c) 572 Turn to page 102

Incorrect.

We are working with base 3. You can tell by the subscript, remember?

Return to page 93 and try again.

Incorrect.

The subscript of 5 indicates that we are working in base 5. Therefore, the number with the exponent on it should be a 5.

Return to page 86 and work that problem again.

No!

We are in base 7. Return to page 103 and try again.

96 is the correct answer!

Now write $4,321_5$ in base 10.

(a) 81 Turn to page 103

(b) 586 Turn to page 105

(c) 211 Turn to page 108

317 is the correct answer!

Now work this one.

$2{,}010_5$ is the number _____ in base 10.

(a) 18 Turn to page 110

(b) 1275 Turn to page 116

(c) 255 Turn to page 105

Your answer is incorrect.

Return to page 97 and begin this last section again.

Very good! You have completed this Unit. Let's review what we have learned.

1. You have learned how to work with integral exponents.

2. You have learned how to write numbers to different bases in expanded exponential form.

3. You have learned how to convert numbers in different bases to base 10.

You are now ready for a test over this Unit. Go tell your teacher you have finished.

What??? How did you get that answer?

Work the problem on page 113 again.

Your answer is incorrect.

Return to page 97 and begin this last section again.

Incorrect.

The subscript tells you what base you are using.

Return to page 102 and work the problem again.

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CAI MATHEMATICS

TEST QUESTIONS

UNIT 20 - CONCEPTS OF NUMBER BASES

1. Write 723 in expanded exponential form

   (a) $700 + 20 + 3$

   (b) $7 \times 100 + 2 \times 10 + 3$

   (c) $7 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$

2. The number $1011_2$ equals _____ in base 10.

   (a) 1011

   (b) 11

   (c) 6

3. Write $11101_2$ in expanded exponential form.

   (a) $1 \times 10^4 + 1 \times 10^3 + 1 \times 10^2 + 0 \times 10^1 + 1 \times 10^0$

   (b) 29

   (c) $1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

4. $10^0 =$

   (a) 1

   (b) 10

   (c) 0

5. Does $4020 = 5 \times 10^3 + 0 \times 10^2 + 2 \times 10^0$?

   (a) Yes

   (b) No

6. Write 1001 in expanded exponential form.

   (a) [illegible] + 1

   (b) [illegible] + 1

   (c) [illegible] $\times 10^3 + 1 \times 10^0$

7. $421_5$ = _____ base 10

(a) 111

(b) 421

(c) 37

8. Which of the following is in expanded exponential form?

(a) 3000 + 400 + 20 + 5

(b) 3 x 1000 + 4 x 100 + 2 x 10 + 5

(c) $3 \times 10^3 + 4 \times 10^2 + 2 \times 10^1 + 5 \times 10^0$

9. $100011_2$ = _____ base 10

(a) 111

(b) 67

(c) 100011

10. Write 25 in expanded exponential form

(a) $2 \times 10^1 + 5 \times 10^0$

(b) 20 + 5

(c) $11001_2$

11. Write $101010_2$ in expanded exponential form.

(a) $2^5 + 2^3 + 2^1$

(b) $1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

(b) 42

12. $111_5$ = _____ base 10

(a) 16

(b) 111

(c) 31

13. Is $10111_2$ in expanded exponential form?

(a) Yes

(b) No

14. $3 \times 5^2 + 2 \times 5^1 + 4 \times 5^0 =$ _____

(a) 324

(b) $324_5$

(c) $324_2$

15. The number $1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$ is written in base _____.

(a) 10

(b) 4

(c) 2

16. Write 3061 in expanded exponential form.

(a) $3 \times 10^3 + 6 \times 10 + 1$

(b) $3 \times 10^2 + 6 \times 10^1 + 1 \times 10^0$

(c) $3000 + 60 + 1$

17. $10010_2$ written in expanded exponential form =

(a) $10000 + 10$

(b) $1 \times 10^4 + 0 \times 10^3 + 0 \times 10^2 + 1 \times 10^1 + 0 \times 10^0$

(c) $1 \times 2^4 + 1 \times 2^1$

18. $111_2$ = _____ in base 10.

(a) 111

(b) 7

(c) 16

19. $3 \times 7^3 + 2 \times 7^1 + 1 \times 7^0 = 3021$ in base _____.

(a) 7

(b) 10

(c) 3

20. What base is 421 written in?

(a) 4

(b) 10

(c) 2

21. $224_5$ = _____ base 10

(a) 244

(b) 10

(c) 74

22. What number is $1 \times 2^5 + 1 \times 2^3 + 1 \times 2$ equal to?

(a) $1011_2$

(b) 101010

(c) $101010_2$

23. $11101_2$ means that 11101 is written in base _____

(a) 5

(b) 2

(c) 10

24. $215$ = _____$_{10}$

(a) 11

(b) 7

(c) 21

25. $110011_2$ written in expanded exponential form is:

(a) $1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

(b) $1 \times 10^5 + 1 \times 10^4 + 0 \times 10^3 + 0 \times 10^2 + 1 \times 10^1 + 1 \times 10^0$

(c) $1 \times 2^4 + 1 \times 2^3 + 1 \times 2^1 + 1 \times 2^0$

Answer Sheet - Unit 20

Concepts of Number Bases

| | | |
|---|---|---|
| 1. | c | 2 |
| 2. | b | 4 |
| 3. | c | 3 |
| 4. | a | 1 |
| 5. | b | 2 |
| 6. | c | 2 |
| 7. | a | 4 |
| 8. | c | 1 |
| 9. | b | 4 |
| 10. | a | 2 |
| 11. | a | 3 |
| 12. | c | 4 |
| 13. | b | 3 |
| 14. | b | 1 |
| 15. | c | 1 |
| 16. | a | 2 |
| 17. | c | 3 |
| 18. | b | 4 |
| 19. | a | 1 |
| 20. | b | 1 |
| 21. | c | 4 |
| 22. | c | 1 |
| 23. | b | 1 |
| 24. | a | 3 |
| 25. | a | 3 |